was eliminated by substitution of the 5-mil teflon. In succeeding experiments, the light emission and sharp pressure rise occurred nearly simultaneously, the light preceding the pressure rise on the average of 0.1 msec. Since the light deflection values can be read more accurately from the phototrace, it is these values that are reported here.

Olin Mathieson technical grade anhydrous hydrazine density 1.004, which was certified by the manufacturer to contain 1.4% water and 0.005% insoluble matter, was used without further purification. Nitrogen tetroxide, having a reported minimum purity of 99.5%, was obtained from the Mathieson Company and was also used directly as supplied.

Results

A summary of the ignition delay data for the hydrazine-nitrogen tetroxide system, both with and without additives, is shown graphically in Fig. 1. The values given in Fig. 1 were obtained by normalizing on an average delay time value for hydrazine. This was necessary because slight differences in delay times were observed for pure hydrazine during the course of the experimental program when it became necessary to change to a new lot. In the graph, the dots represent the average delay times and the horizontal lines the average errors for each set of determinations. In most cases, the delay time was obtained as the average of three separate measurements. The error limits are small enough to establish that the additives are effecting real changes in the delay times. The large spread of values obtained in the runs with carbontetrabromide is attributed to chemical reaction with the hydrazine, since a pressure build-up in the hydrazine storage indicated a slow reaction with carbontetrabromide.

With the exception of iodine, which was added to the nitrogen tetroxide, all additives were dissolved in the hydrazine. The numbers in parenthesis in Fig. 1 indicate the concentration of additive in weight percent. In a few cases, indicated by (*s*) in Fig. 1, the additive was only partially soluble in the hydrazine. In these cases, 1% of additive was placed in the hydrazine, the liquid was agitated until no more additive appeared to be dissolving, and the resulting saturated hydrazine solution was then decanted from the remaining undissolved additive.

The fact that delay times were shortened with surfactant FC-126 and Santomerse 85, two surface active agents, indicates that solubility of the two reactants is a factor in the delay times. Two possible intermediates in the reaction, hydrazine nitrate and ammonium nitrate, also reduced the ignition time.

In one set of experiments the reactor was flushed with N_2O_4 instead of nitrogen, and the delay time for N_2H_4-N_2O_4 without additives was measured. As expected, a significant decrease in the ignition delay time was observed. These results are in agreement with those observed for the triethylamine-nitric acid reaction,[5] where the delay times became shorter when the initial gas-phase oxidant concentration (O_2 or N_2O_4) was increased. It is reported,[5] in fact, that shortest delay times were obtained when nitrogen tetroxide was substituted for air in the bomb reactor. Acceleration by gas-phase oxidant, coupled with the fact that most gas-phase reactions involve free radical mechanisms, makes us believe that the hydrazine-nitrogen tetroxide reaction proceeds by a free radical mechanism, although the delay reported here does not conclusively support this view.

References

1 Kilpatrick, M. and Baker, L. L., Jr., *Fifth Symposium on Combustion* (Reinhold Publishing Corp., New York, 1955), Paper 11, p. 196.

2 Skinner, G. B., Hedley, W. H., and Snyder, A. D., "Mechanism and chemical inhibition of the hydrazine-nitrogen tetroxide reaction," Monsanto Research Corp., Dayton, Ohio, Tech. Doc. Rept. Aeronautical Systems Div., TDR-62-1041 (December 1962).

3 Friedman, R., Barnes, W. P., Jr., and Markels, M., Jr., "A study of explosions induced by contact of hydrazine-type fuels with nitrogen tetroxide," Atlantic Research Corp., Alexandria, Va., Tech. Doc. Rept. Aeronautical Systems Div., TDR-62-685 (September 1962).

4 Weiss, H. G. and Klusmann, E. B., "Determination of miscibility characteristics of highly reactive liquids," J. Am. Chem. Soc. **84**, 4993 (1962).

5 Schalla, R. L. and Fletcher, E. A., *Sixth Symposium on Combustion* (Reinhold Publishing Corp., New York, 1957), Paper 123, p. 911.

Payload Scaling Laws for Boost Rockets

DONALD VAN ZELM WADSWORTH*

Bell Telephone Laboratories, Inc., Whippany, N. J.

Explanatory

FOR a given change in payload weight, we wish to determine what compensatory changes (i.e., scaling) must be made in other rocket parameters (such as thrust and initial stage weights) in order that the final burnout position, velocity, and time be unaffected. This is a problem of interest in preliminary design studies when numerical data generated for one payload are to be applied to other payloads. In this article, we determine the scaling laws that leave a boost rocket trajectory invariant to payload weight changes. These laws are different from the usual "payload exchange ratios" used in mission design where only the characteristic velocity is held invariant.

Derivation of Scaling Laws

The rocket parameters appearing explicitly in the differential equations of motion must be invariant to changes in payload weight if the trajectory is to be unaltered. The invariant parameters can be found by examining the equations of motion:

$$\mathbf{a} = (F/m)\mathbf{r} - (D/m)\mathbf{r} - \mathbf{g}$$

where $\mathbf{a}$ is the acceleration, $\mathbf{r}$ a unit vector parallel to the rocket's longitudinal axis, $\mathbf{g}$ the acceleration of gravity, m the mass, F the thrust, and D the drag force. We assume thrust and drag act parallel to the longitudinal axis. We neglect lift for the present discussion, since angles of attack are small during the gravity turn. This will not affect the generality of the scaling laws. Boldface characters denote vector quantities.

The thrust[1] consists of a momentum thrust F_m and a pressure thrust F_p:

$$F_m = p_c r^{-1} a f(\gamma, r)$$

$$F_p = a(p_e - p)$$

where p_c is the design chamber pressure of the rocket motor, p_e the pressure at the exit of the nozzle, p the ambient pressure (a function of altitude), γ the ratio of the specific heats of the exhaust gases, r the ratio of the nozzle area at the exit to the area at the throat, a the cross-sectional area of the nozzle exit, and $f(\ )$ denotes "function of." For scaling, we shall assume that the parameters γ, p_c, p_e, r are invariant. It turns out that vacuum specific impulse (a property of the propellant) must be an invariant, so that it is reasonable to assume γ to be invariant. The only thrust parameter to be scaled is a.

The drag force is given by

$$D = \tfrac{1}{2}\rho V^2 A C_D$$

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* Member of Technical Staff. Member AIAA.

where ρ is atmospheric density, V the speed relative to the atmosphere, A the effective cross-sectional area of the rocket, and C_D the coefficient of drag (which depends on Mach number primarily). We shall assume that C_D is invariant to scaling so that the only drag parameter to be scaled is A.

If we let μ be the ratio of the final to initial weight of a single stage rocket, t the burning time, and s the time from initiation of burning, then mass m is given by

$$m = h(t - s + \mu s)(1 - \mu)^{-1}$$

where h is the mass burning rate (assumed independent of s). Reference to the equations of motion shows that they will be invariant to payload changes if the parameters t, μ, a/h, and A/h are kept invariant (in addition to those specified previously). Under these conditions, it is evident that for any given value of p the function $Fh^{-1}g^{-1} = I$ = specific impulse will be invariant as stated previously (g is the mass-to-weight conversion factor). From the preceding definitions, we have $gh = t^{-1}W(1 - \mu)$, where W is the initial weight. Then the invariants a/h and A/h can be replaced by the invariants a/W and A/W.

For an n stage rocket, let W_L denote payload weight, w_i the initial weight of the ith stage (without any upper stages), W_i the initial weight of the ith rocket (weight of ith stage plus all upper stages and payload), I_i the specific impulse of the ith stage, σ_i the final to initial weight ratio of the ith stage, and μ_i the final to initial weight ratio of the ith rocket.

By definition,

$$\mu_i = \frac{W_{i+1} + \sigma_i w_i}{W_i} = 1 - (1 - \sigma_i)\frac{w_i}{W_i}$$

$$W_i = w_i + w_{i+1} + \ldots w_n + W_L$$

If Δ denotes a change in a quantity, we obtain from the preceding the following scaling laws:

$$(1 - \mu_i)\Delta W_i = \Delta w_i - \Delta(\sigma_i w_i)$$

$$\Delta a_i = (a_i/W_i)\Delta W_i \qquad \Delta A_i = (A_i/W_i)\Delta W_i$$

$$\Delta F_i = (F_i/a_i)\Delta a_i$$

In general, the stage weight at burnout can be expressed as

$$\sigma_i w_i = \omega_{xi} + \alpha_i w_i + \beta_i F_i$$

which states that the burnout weight contains a portion ω_{xi} independent of stage weight and thrust, a portion $\alpha_i w_i$ dependent upon the stage weight (fuel tanks and structure), and a portion $\beta_i F_i$ dependent upon the thrust level (the rocket motor and pumps). Consequently, if we assume ω_{xi}, α_i, and β_i are not to be scaled, $\Delta(\sigma_i w_i) = \alpha_i \Delta w_i + \beta_i \Delta F_i$. For a wide variety of rockets, it can be assumed that $\alpha_i \simeq 0.1$ and $\beta_i \simeq 0.01$.

Scaling Laws for a Two-Stage Rocket

Using the preceding scaling laws and the invariance of t_i, μ_i, ω_{xi}, α_i, and β_i, we obtain for a two-stage rocket (in which one stage is in the atmosphere)

$$\Delta I_1 = \Delta I_2 = 0$$

$$\Delta F_1 = (F_1/W_1)\Delta W_1 \qquad \Delta A_1 = (A_1/W_1)\Delta W_1$$

$$\Delta a_1 = (a_1/W_1)\Delta W_1$$

$$\Delta F_2 = (F_2/W_2)\Delta W_2 \qquad \Delta a_2 = (a_2/W_2)\Delta W_2$$

$$\Delta W_1 = \Delta w_1 + \Delta W_2 \qquad \Delta W_2 = \Delta w_2 + \Delta W_L$$

$$\Delta w_1 = \left(1 - \mu_1 + \frac{\beta_1 F_1}{W_1}\right)\left(\mu_1 - \alpha_1 - \frac{\beta_1 F_1}{W_1}\right)^{-1} \times (\Delta w_2 + \Delta W_L)$$

$$\Delta w_2 = \left(1 - \mu_2 + \frac{\beta_2 F_2}{W_2}\right)\left(\mu_2 - \alpha_2 - \frac{\beta_2 F_2}{W_2}\right)^{-1} \Delta W_L$$

Using $-\dot{w}$ to denote weight rate of propellant consumption ($\dot{w}$ is a negative quantity), we also have

$$\Delta(-\dot{w}_1) = (1 - \mu_1)t_1^{-1}\Delta W_1$$

$$\Delta(-\dot{w}_2) = (1 - \mu_2)t_2^{-1}\Delta W_2$$

If the $\dot{w}_i$ are the design parameters rather than the σ_i, it is convenient to calculate μ_i from $\mu_i = 1 + \dot{w}_i t_i/W_i$.

Example

The preceding scaling laws were applied to a typical two-stage ICBM using a realistic physical model of the rocket and its environment, including a rotating oblate earth with a standard atmosphere and variation of the coefficients of lift and drag with Mach number. A nominal booster power flight trajectory for a 4500-lb payload was simulated on the digital computer. Then the payload was reduced by 700 lb and the scaling laws used to determine the following changes in the booster parameters:

$$\Delta F_1 = -32{,}750 \text{ lb} \qquad \Delta F_2 = -6706 \text{ lb}$$
$$\Delta A_1 = -8.6 \text{ ft}^2 \qquad \Delta a_1 = -334 \text{ in.}^2$$
$$\Delta \dot{w}_1 = 131 \text{ lb/sec} \qquad \Delta \dot{w}_2 = 21 \text{ lb/sec}$$
$$\Delta W_1 = -24{,}472 \text{ lb} \qquad \Delta W_2 = -4167 \text{ lb}$$
$$\Delta w_1 = -20{,}304 \text{ lb} \qquad \Delta w_2 = -3493 \text{ lb}$$

With the new payload, the "scaled" booster trajectory was simulated and, as expected, was identical with the nominal trajectory. For this example we took $\mu_1 = \mu_2 = 0.26$.

Reference

[1] Koelle, H. H., *Handbook of Astronautical Engineering* (McGraw-Hill Book Co., Inc., New York, 1961), pp. 20-1–20-9.

Unsteady Flow Past Junctures in Ducts

A. G. Hammitt* and H. J. Carpenter†
TRW Space Technology Laboratories, Redondo Beach, Calif.

The problem of shock waves in ducts is one that is important to bomb shelter installations that use some sort of ventilation or other piping system connecting to the atmosphere. Although these systems are relatively simple from a plumbing point of view, they are generally beyond the state of the art of standard one-dimensional flow analysis. In particular, they usually contain L and T junctures for which standard one-dimensional flow analyses are not available.

The purpose of this note is to set up the boundary conditions to be applied at such discontinuities to fit into a one-dimensional unsteady flow analysis. The detailed, non-one-dimensional flow pattern near the juncture will be ignored, but the effect a few diameters away, where the flow has become one-dimensional again, will be described. The viscous and shock losses that occur near the junctures will be summed up by (over-all) loss coefficients. These loss coefficients can be obtained from steady flow data and used to predict the losses for the unsteady cases. Although the treatment of the flow at the junctures is empirical with regard to these loss coefficients, it is the best that can be expected within the framework of one-dimensional inviscid theory. A true de-

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* Manager, Aerosciences Laboratory Research Staff. Member AIAA.
† Member of Technical Staff. Member AIAA.